Subject CM1 Formula Sheet

Introduction

This document summarises some of the important formulae required in the Subject CM1 exam. This document is not an exhaustive list. Students ought to be able to evaluate assurances and annuities through integration methods where appropriate. The formulae associated with these integration methods are not provided here.

Space has been left in the margins allowing students to add their own notes as studies progress.

Where appropriate, formulae are provided referencing the three tables in the Formulae and Tables for Actuarial Examinations:

- ELT15, page 67
- AM92, page 73
- PMA92 and PFA92, page 109

Note that the mortality tables listed above do not include the same information. *Generally speaking* ELT15 and AM92 are used for single life annuities / assurances, where PMA92 and PFA92 are used for joint life annuities/assurances. As students progress through their studies, other smaller nuances with regards to the tables ought to become clear.

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0 Financial mathematics basics

0.1 Rates of interest / discount / force of interest

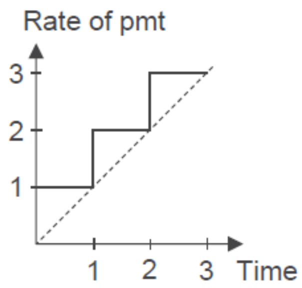
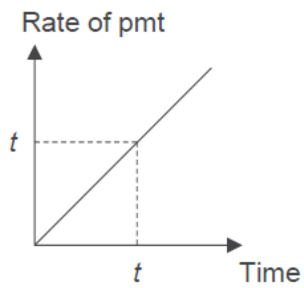
Symbol	Meaning	Calculation Method
i	Effective rate of interest (i may also be used for simple interest)	n/a
$i^{(p)}$	Rate of interest convertible pthly. This is not an effective rate	$i^{(p)} = p[(1+i)^{1/p} - 1]$
v	Effective discount factor	$\left(\frac{1}{1+i}\right)$ or $(1-d)$
d	Effective rate of discount	$\left(\frac{i}{1+i}\right)$ or $(1-v)$ or (iv)
$d^{(p)}$	Rate of discount convertible pthly. This is not an effective rate	$d^{(p)} = p[1-(1+i)^{-1/p}]$
δ	Force of interest	$Ln(1+i)$

0.2 Level annuity certain

Symbol	Meaning	Calculation Method
$a_{\overline{n}\rceil}$	PV of annuity certain payable in arrears *Present value of payments of 1 made at the end of each year for n years*	$\frac{1-v^n}{i}$
$\ddot{a}_{\overline{n}\rceil}$	PV of annuity certain payable in advance *Present value of payments of 1 made at the start of each year for n years*	$\frac{1-v^n}{d} = (1+i)a_{\overline{n}\rceil}$
$\bar{a}_{\overline{n}\rceil}$	PV of annuity certain payable continuously *Present value of payments of 1 made continuously each year for n years*	$\frac{1-v^n}{\delta}$
$s_{\overline{n}\rceil}$	AV of annuity certain payable in arrears *Accumulated value of payments of 1 made at the end of each year for n years*	$\frac{(1+i)^n - 1}{i} = (1+i)^n a_{\overline{n}\rceil}$
$\ddot{s}_{\overline{n}\rceil}$	AV of annuity certain payable in advance *Accumulated value of payments of 1 made at the start of each year for n years*	$\frac{(1+i)^n - 1}{d}$
$\bar{s}_{\overline{n}\rceil}$	AV of annuity certain payable continuously *Accumulated value of payments of 1 made continuously each year for n years*	$\frac{(1+i)^n - 1}{\delta}$

Note: when there is no possibility of confusion with a life annuity, the term ‘annuity’ can be used as an alternative to annuity certain.

0.3 Increasing annuity certain

Symbol	Meaning	Calculation Method
$(Ia)_{\overline{n\rvert}}$	PV of an increasing annuity certain payable in arrears *Present value of increasing payments made at the end of each year for n years*	$\dfrac{\ddot{a}_{\overline{n\rvert}} - nv^n}{i}$
$(I\ddot{a})_{\overline{n\rvert}}$	PV of an increasing annuity certain payable in advance *Present value of increasing payments made at the start of each year for n years*	$\dfrac{\ddot{a}_{\overline{n\rvert}} - nv^n}{d}$
$(I\bar{a})_{\overline{n\rvert}}$	PV of an increasing annuity certain payable continuously throughout the year *Present value of increasing payments, where the payment is made continuously throughout the year, for n years* 	$\dfrac{\ddot{a}_{\overline{n\rvert}} - nv^n}{\delta}$
$(\bar{I}\bar{a})_{\overline{n\rvert}}$	PV of a continuously increasing annuity certain continuously throughout the year *Present value of a continuously increasing payment, where the payment is made continuously throughout the year, for n years* 	$\dfrac{\bar{a}_{\overline{n\rvert}} - nv^n}{\delta}$

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0.4 Pthly payable annuity certain

Symbol	Meaning	Calculation Method	
$a_{\overline{n}\rvert}^{(p)}$	PV of annuity certain payable pthly in arrears *Present value of payments of 1 pa, payable pthly in arrears*	$\dfrac{1-v^n}{i^{(p)}}$	Where: $\left(1+\dfrac{i^{(p)}}{p}\right)^p=(1+i)$
$s_{\overline{n}\rvert}^{(p)}$	AV of annuity certain payable pthly in arrears *Accumulated value of payments of 1 pa, payable pthly in arrears*	$\dfrac{(1+i)^n-1}{i^{(p)}}$	$i^{(p)}=p\left[(1+i)^{1/p}-1\right]$
$\ddot{a}_{\overline{n}\rvert}^{(p)}$	PV of annuity certain payable pthly in advance *Present value of payments of 1 pa, payable pthly in advance*	$\dfrac{1-v^n}{d^{(p)}}$	Where: $\left(1-\dfrac{d^{(p)}}{p}\right)^p=(1-d)$
$\ddot{s}_{\overline{n}\rvert}^{(p)}$	AV of annuity certain payable pthly in advance *Accumulated value of payments of 1 pa, payable pthly in advance*	$\dfrac{(1+i)^n-1}{d^{(p)}}$	$d^{(p)}=p\left[1-(1-d)^{1/p}\right]$ $=p\left[1-(1+i)^{-1/p}\right]$

1 Life probabilities

Symbol	Meaning	Calculation Method
${}_tq_x$	Probability that a life aged *x* dies in the next *t* years	${}_tq_x = 1 - \frac{l_{x+t}}{l_x} = \frac{l_x - l_{x+t}}{l_x}$
q_x	Probability that a life aged *x* dies in the next 1 year	$q_x = d_x / l_x$
${}_tp_x$	Probability that a life aged *x* survives the next *t* years	${}_tp_x = \frac{l_{x+t}}{l_x}$
p_x	Probability that a life aged *x* survives the next 1 year	$1 - q_x$ or use $p_x = l_{x+1} / l_x$
${}_{n\|t}q_x$	Probability that a life aged *x* survives the next *n* years, but then dies in the subsequent *t* years	${}_np_x \times {}_tq_{x+n}$ $= {}_np_x - {}_{n+t}p_x$ or ${}_{n+t}q_x - {}_nq_x$ $= \frac{l_{x+n} - l_{x+n+t}}{l_x}$
${}_{n\|}q_x$	Probability that a life aged *x* survives the next *n* years, but then dies in the subsequent 1 year	${}_np_x \times q_{x+n}$ $= \frac{l_{x+n} - l_{x+n+1}}{l_x} = \frac{d_{x+n}}{l_x}$

2 Assurances

2.1 Level Life Assurances (payments at end of the year)

Symbol	Meaning	Calculation method
A_x	Whole of Life *Lump sum paid at end of year of death of a life aged x* exact	Using AM92, tabulated at 4% and 6% Using PMA92/PFA92 then $A_x = 1 - d\ddot{a}_x$
$A^1_{x:\overline{n}\rceil}$	Term Assurance *Lump sum at end of year of death of a life aged x* exact, *only if life dies within n years*	$A^1_{x:\overline{n}\rceil} = A_{x:\overline{n}\rceil} - A_{x:\overline{n}\rceil}^{\ \ 1}$ $= A_{x:\overline{n}\rceil} - v^n\,{}_np_x$ Using AM92, if x+n=60 or 65, then $A_{x:\overline{n}\rceil}$ is tabulated at 4% and 6%. If not, then: $A_x - v^n\,{}_np_x A_{x+n}$ Using PMA92/PFA92 then use formulas as above, calculating $A_x = 1 - d\ddot{a}_x$
$A_{x:\overline{n}\rceil}^{\ \ 1}$	Pure Endowment *Lump sum at end of term if a life aged x exact survives term of n years*	Using AM92, and PMA92/PFA92: $v^n\,{}_np_x$ Using AM92 at 4%: D_{x+n}/D_x
$A_{x:\overline{n}\rceil}$	Endowment Assurance *Lump sum at end of year of death of life aged x* exact, *if life dies within n years. If life survives, then payment at time n*	$A^1_{x:\overline{n}\rceil} + A_{x:\overline{n}\rceil}^{\ \ 1}$ $= A_x - v^n\,{}_np_x A_{x+n} + v^n\,{}_np_x$ Using AM92, if x+n=60 or 65, then $A_{x:\overline{n}\rceil}$ is tabulated at 4% and 6% Using PMA92/PFA92 then use formulas as above, calculating $A_x = 1 - d\ddot{a}_{x:\overline{n}\rceil}$
${}_{n\mid}A_x$	Deferred whole of Life *Lump sum paid at end of year of death of life aged x* exact, *provided life survives first n years.*	$v^n\,{}_np_x A_{x+n}$
${}_{m\mid}A^1_{x:\overline{n}\rceil}$	Deferred Term Assurance *Lump sum at end of year of death of life aged x* exact, *only if life dies within n years, starting after m years, provided life survives first m years*	$v^m\,{}_mp_x A^1_{x+m:\overline{n}\rceil}$

2.2 Level Life Assurances (payable immediately)

Symbol	Meaning	Calculation method
$\bar{A}_x$	Whole of Life *Lump sum paid immediately on death of x*	$\bar{A}_x \simeq (1+i)^{0.5} A_x$ Using A_x formula on previous page $\bar{A}_x \simeq 1-\delta \bar{a}_x$
$\bar{A}^1_{x:\overline{n}\rceil}$	Term Assurance *Lump sum paid immediately on death of x, only if life within n years*	$\bar{A}^1_{x:\overline{n}\rceil} \simeq (1+i)^{0.5} A^1_{x:\overline{n}\rceil}$ Using $A^1_{x:\overline{n}\rceil}$ formula on previous page
$\bar{A}_{x:\overline{n}\rceil}$	Endowment Assurance *Lump sum paid immediately on death of x, if life dies within n years. If life survives, then payment at time n*	$\bar{A}^1_{x:\overline{n}\rceil} + A_{x:\overline{n}\rceil}^{\ \ 1} \simeq (1+i)^{0.5} A^1_{x:\overline{n}\rceil} + v^n\ {}_np_x$ Using $A^1_{x:\overline{n}\rceil}$ formula on previous page $\bar{A}_{x:\overline{n}\rceil} \simeq 1-\delta \bar{a}_{x:\overline{n}\rceil}$
${}_{n\vert}\bar{A}_x$	Deferred whole of Life *Lump sum paid immediately on death of x, provided life survives first n years.*	$v^n\ {}_np_x (1+i)^{0.5} A_{x+n}$
${}_{m\vert}\bar{A}^1_{x:\overline{n}\rceil}$	Deferred Term Assurance *Lump sum paid immediately on death of x, only if life dies within n years, starting after m years, provided life survives first m years*	$v^m\ {}_mp_x \bar{A}^1_{x+m:\overline{n}\rceil}$

2.3 Simple Increasing Life Assurances (payable at end of the year of death)

Symbol	Meaning	Calculation method
$(IA)_x$	Whole of Life *Increasing lump sum paid at the end of year of death of x*	Using AM92, this is tabulated at 4% and 6%
$(IA)^1_{x:\overline{n}\rceil}$	Term Assurance *Increasing lump sum paid at the end of year of death of x, only if life within n years*	Using AM92 at 4% and 6% $(IA)_x - v^n\ {}_np_x \left[(IA)_{x+n} + nA_{x+n}\right]$
$(IA)_{x:\overline{n}\rceil}$	Endowment Assurance *Increasing lump sum paid at the end of year of death of x, if life dies within n years. If life survives, then payment of n at time n*	Using AM92 at 4% and 6%: $(IA)^1_{x:\overline{n}\rceil} + nA_{x:\overline{n}\rceil}^{\ \ 1}$ $= (IA)_x - v^n\ {}_np_x \left[(IA)_{x+n} + nA_{x+n}\right]$ $+ nv^n\ {}_np_x$
${}_{n\vert}(IA)_x$	Deferred Whole of Life *Increasing lump sum paid at the end of year of death of x, provided life survives first n years.*	$v^n\ {}_np_x (IA)_{x+n}$

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2.4 Simple Increasing Life Assurances (payable immediately on death)

Symbol	Meaning	Calculation method
$(I\bar{A})_x$	Whole of Life *Increasing lump sum paid immediately on death of x*	Using AM92 at 4% and 6% $\simeq (1+i)^{0.5}(IA)_x$
$(I\bar{A})^1_{x:\overline{n}\vert}$	Term Assurance *Increasing lump sum paid immediately on death of x, only if life within n years*	Using AM92 at 4% and 6% $\simeq (1+i)^{0.5}(IA)^1_{x:\overline{n}\vert}$ $= (1+i)^{0.5}\left[(IA)_x - v^n\,{}_np_x\left((IA)_{x+n} + nA_{x+n}\right)\right]$
$(I\bar{A})_{x:\overline{n}\vert}$	Endowment Assurance *Increasing lump sum paid immediately on death of x, if life dies within n years. If life survives, then payment of n at time n*	Using AM92 at 4% and 6%: $\simeq (1+i)^{0.5}(IA)^1_{x:\overline{n}\vert} + nv^n\,{}_np_x$ $= (1+i)^{0.5}\left[(IA)_x - v^n\,{}_np_x\left((IA)_{x+n} + nA_{x+n}\right)\right]$ $+ nv^n\,{}_np_x$
${}_{n\vert}(IA)_x$	Deferred Whole of Life *Increasing lump sum paid immediately on death of x, provided life survives first n years.*	$v^n\,{}_np_x(I\bar{A})_{x+n}$

3 Annuities

3.1 Level Life Annuities (annual payments at start / end of year)

Symbol	Meaning	Calculation method
$\ddot{a}_x$	Whole life immediate annuity-due *Regular payments in advance as long as life is alive*	Using AM92, tabulated at 4% and 6% Using PMA92/PFA92, tabulated at 4%
a_x	Whole life immediate annuity *Regular payments in arrears as long as live is alive*	$\ddot{a}_x - 1$
$\ddot{a}_{x:\overline{n}\rceil}$	Temporary immediate annuity-due *Regular payments in advance during term as long as life is alive, up to n years*	Using AM92, if x+n=60 or 65, then $\ddot{a}_{x:\overline{n}\rceil}$ is tabulated at 4% and 6% If not, then using AM92 or PMA92/PFA92, can be calculated: $\ddot{a}_x - v^n\,{}_np_x\ddot{a}_{x+n}$
$a_{x:\overline{n}\rceil}$	Temporary immediate annuity *Regular payments in in arrears during term as long as life is alive, up to n years*	Using AM92, if x+n=60 or 65, then $a_{x:\overline{n}\rceil}$ can be calculated as: $\ddot{a}_{x:\overline{n}\rceil} - (1 - v^n\,{}_np_x)$ If not, then using AM92 or PMA92/PFA92, can be calculated: $a_x - v^n\,{}_np_x a_{x+n}$ $= (\ddot{a}_x - 1) - v^n\,{}_np_x(\ddot{a}_{x+n} - 1)$
$\ddot{a}_{\overline{x:\overline{n}\rceil}}$	Guaranteed annuity-due *Whole of life immediate annuity-due with n years guaranteed*	$\ddot{a}_{\overline{n}\rceil} + v^n\,{}_np_x\,\ddot{a}_{x+n}$
$a_{\overline{x:\overline{n}\rceil}}$	Guaranteed annuity *Whole of life immediate annuity in arrears with n years guaranteed*	$a_{\overline{n}\rceil} + v^n\,{}_np_x\,a_{x+n}$
${}_{n\vert}\ddot{a}_x$	Deferred whole life annuity-due *Regular payments in advance as long as life is alive, provided life survives first n years.*	$v^n\,{}_np_x\,\ddot{a}_{x+n}$
${}_{n\vert}a_x$	Deferred whole life annuity *Regular payments in arrears as long as life is alive, provided life survives first n years.*	$v^n\,{}_np_x\,a_{x+n}$

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3.2 Level Life Annuities (continuous annual payments)

Symbol	Meaning	Calculation method
$\bar{a}_x$	Whole life continuous immediate annuity *Continuous payments as long as life is alive*	$\simeq \ddot{a}_x - \frac{1}{2}$
$\bar{a}_{x:\overline{n}\rvert}$	Temporary continuous immediate annuity *Continuous payments made as long as life is alive, up to n years*	Using AM92, if x+n=60 or 65, then $\bar{a}_{x:\overline{n}\rvert}$ can be calculated as: $\simeq \ddot{a}_{x:\overline{n}\rvert} - \frac{1}{2}(1 - v^n\ {}_n p_x)$ If not, then using AM92 or PMA92/PFA92, can be calculated: $\simeq \ddot{a}_x - v^n\ {}_n p_x \ddot{a}_{x+n} - \frac{1}{2}(1 - v^n\ {}_n p_x)$
$\bar{a}_{\overline{x:\overline{n}\rvert}}$	Guaranteed continuous immediate annuity *Continuous payments as long as life is alive, with n years guaranteed*	$\bar{a}_{\overline{n}\rvert} + v^n\ {}_n p_x\ \bar{a}_{x+n}$
${}_{n\rvert}\bar{a}_x$	Deferred whole life continuous annuity *Continuous payments as long as life is alive, provided life survives first n years.*	$v^n\ {}_n p_x\ \bar{a}_{x+n}$

3.3 Simple Increasing Life Annuities (annual payments at start / end of year)

Symbol	Meaning	Calculation method
$(I\ddot{a})_x$	Increasing life immediate annuity-due *Increasing regular payments in advance as long as life is alive*	Using AM92, this is tabulated at 4% and 6%
$(Ia)_x$	Increasing life immediate annuity *Increasing regular payments in arrears as long as live is alive*	$(I\ddot{a})_x - \ddot{a}_x$
$(I\ddot{a})_{x:\overline{n}\rvert}$	Increasing temporary immediate annuity-due *Increasing regular payments in advance during term as long as life is alive, up to n years*	$(I\ddot{a})_x - v^n\ {}_np_x[(I\ddot{a})_{x+n} + n\ddot{a}_{x+n}]$
$(Ia)_{x:\overline{n}\rvert}$	Increasing temporary immediate annuity *Increasing regular payments in arrears during term as long as life is alive, up to n years*	$(Ia)_x - v^n\ {}_np_x[(Ia)_{x+n} + na_{x+n}]$
${}_{n\rvert}(I\ddot{a})_x$	Deferred increasing life annuity-due *Increasing regular payments in advance as long as life is alive, provided life survives first n years*	$v^n\ {}_np_x\ (I\ddot{a})_{x+n}$
${}_{n\rvert}(Ia)_x$	Deferred increasing life annuity *Increasing regular payments in arrears as long as life is alive, provided life survives first n years*	$v^n\ {}_np_x\ (Ia)_{x+n}$

3.4 Simple Increasing Life Annuities (continuous annual payments)

Symbol	Meaning	Calculation method
$(I\bar{a})_x$	Increasing life immediate continuous annuity *Increasing regular payments payable continuously as long as life is alive*	$\simeq (I\ddot{a})_x - \frac{1}{2}\ddot{a}_x$
$(I\bar{a})_{x:\overline{n}\rvert}$	Increasing temporary immediate continuous annuity *Increasing regular payments payable continuously as long as live is alive, up to n years*	$(I\bar{a})_x - v^n\ {}_np_x\left[(I\bar{a})_{x+n} + n\bar{a}_{x+n}\right]$
${}_{n\rvert}(I\bar{a})_x$	Deferred increasing life continuous annuity *Increasing regular payments payable continuously as long as life is alive, provided life survives first n years*	$v^n\ {}_np_x\ (I\bar{a})_{x+n}$

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3.5 Annuities payable non-annually

Note that some of the below formulae can be seen on page 36 of Formulae and Tables for Actuarial Examinations.

Symbol	Meaning	Calculation method
$\ddot{a}_x^{(p)}$	Whole life immediate annuity-due payable pthly *Regular payments payable pthly in advance as long as life is alive*	$\simeq \ddot{a}_x - \frac{(p-1)}{2p}$
$a_x^{(p)}$	Whole life immediate annuity payable pthly *Regular payments payable pthly in arrears as long as life is alive*	$\simeq a_x + \frac{(p-1)}{2p} = \ddot{a}_x - \frac{(p+1)}{2p}$
$\ddot{a}_{x:\overline{n}\rceil}^{(p)}$	Temporary immediate annuity-due payable pthly *Regular payments payable pthly in advance during term as long as life is alive, up to n years*	Using AM92, if x+n=60 or 65, then $\ddot{a}_{x:\overline{n}\rceil}$ is tabulated at 4% and 6% $\simeq \ddot{a}_{x:\overline{n}\rceil} - \frac{p-1}{2p}\left(1 - v^n \, {}_np_x\right)$ If not, then using AM92 or PMA92/PFA92, can be calculated: $\ddot{a}_{x:\overline{n}\rceil}^{(p)} = \ddot{a}_x^{(p)} - v^n \, {}_np_x \, \ddot{a}_{x+n}^{(p)}$
$a_{x:\overline{n}\rceil}^{(p)}$	Temporary immediate annuity payable pthly *Regular payments payable pthly in arrears during term as long as life is alive, up to n years*	Using AM92, if x+n=60 or 65, then $\ddot{a}_{x:\overline{n}\rceil}$ is tabulated at 4% and 6% $\simeq \ddot{a}_{x:\overline{n}\rceil} - \frac{p+1}{2p}\left(1 - v^n \, {}_np_x\right)$ If not, then using AM92 or PMA92/PFA92, can be calculated: $a_{x:\overline{n}\rceil}^{(p)} = a_x^{(p)} - v^n \, {}_np_x \, a_{x+n}^{(p)}$
$\ddot{a}_{\overline{x:\overline{n}\rceil}}^{(p)}$	Guaranteed annuity-due payable pthly *Whole of life immediate annuity-due payable pthly with n years guaranteed*	$\ddot{a}_{\overline{n}\rceil}^{(p)} + v^n \, {}_np_x \, \ddot{a}_{x+n}^{(p)}$
$a_{\overline{x:\overline{n}\rceil}}^{(p)}$	Guaranteed annuity payable pthly *Whole of life immediate annuity in arrears payable pthly with n years guaranteed*	$a_{\overline{n}\rceil}^{(p)} + v^n \, {}_np_x \, a_{x+n}^{(p)}$
${}_{n\mid}\ddot{a}_x^{(p)}$	Whole life deferred annuity-due payable pthly *Regular payments payable pthly in advance as long as life is alive, provided life survives first n years*	$v^n \, {}_np_x \, \ddot{a}_{x+n}^{(p)}$
${}_{n\mid}a_x^{(p)}$	Whole life deferred annuity payable pthly *Regular payments payable pthly in arrears as long as life is alive, provided life survives first n years*	$v^n \, {}_np_x \, a_{x+n}^{(p)}$

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4 Joint Life probabilities

Symbol	Meaning	Calculation Method
${}_tq_{xy}$	Probability that the joint life status fails within *t* years, *ie* the probability that at least one of *x* and *y* dies in next *t* years	$1 - {}_tp_x \, {}_tp_y$
${}_tp_{xy}$	Probability that the joint life status is still active in *t* years' time, *ie* the probability that both *x* and *y* survive for at least t years	${}_tp_x \, {}_tp_y$
${}_tq_{\overline{xy}}$	Probability that the last survivor status fails within *t* years, *ie* the probability that both *x* and *y* die in next *t* years	${}_tq_x \, {}_tq_y$
${}_tp_{\overline{xy}}$	Probability that the last survivor status is still active in *t* years' time, *ie* the probability that at least one of *x* and *y* survive for at least *t* years	$1 - {}_tq_x \, {}_tq_y$ $= {}_tp_x + {}_tp_y - {}_tp_x \, {}_tp_y$
${}_tq^{\;1}_{xy}$	Probability that life *x* dies first in the next *t* years	Assuming same age and mortality: ${}_tq^{\;1}_{xx} = \frac{1}{2} {}_tq_{xx}$
${}_tq^{\;2}_{xy}$	Probability that life *x* dies second in the next *t* years	Assuming same age and mortality: ${}_tq^{\;2}_{xx} = \frac{1}{2} {}_tq_{\overline{xx}} = \frac{1}{2}\left({}_tq_x\right)^2$

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5 Joint Life Assurances

Symbol	Meaning	Calculation Method
$A_{x:y}$	Joint whole of life assurance *Assurance payments made at the end of year of first death, whenever that occurs*	$1-d\ddot{a}_{x:y}$
$\bar{A}_{x:y}$	Joint whole of life assurance payable immediately *Assurance payments made immediately on the first death, whenever that occurs*	$1-\delta\bar{a}_{x:y}$ or $(1+i)^{0.5}\left(1-d\ddot{a}_{x:y}\right)$
$A_{\overline{x:y}}$	Last survivor whole of life assurance *Assurance payments made at the end of year of second death, whenever that occurs*	$A_x + A_y - A_{x:y}$ or $1-d\ddot{a}_{\overline{x:y}}$
$\bar{A}_{\overline{x:y}}$	Last survivor whole of life assurance payable immediately *Assurance payment made immediately on second death, whenever that occurs*	$\bar{A}_x + \bar{A}_y - \bar{A}_{x:y}$ or $1-\delta\bar{a}_{\overline{x:y}}$
$A_{\overset{1}{x:y}:\overline{n}\rceil}$	Joint life n-year term assurance *Assurance payment made at the end of year of the first death, provided that occurs within the next n years*	$A_{x:y} - v^n\, {}_np_x\, {}_np_y A_{x+n:y+n}$
$\bar{A}_{\overset{1}{x:y}:\overline{n}\rceil}$	Joint life n-year immediately payable term assurance *Assurance payment made immediately on the first death, provided that occurs within the next n years*	$\bar{A}_{x:y} - v^n\, {}_np_x\, {}_np_y \bar{A}_{x+n:y+n}$
$A_{x:y:\overset{1}{\overline{n}\rceil}}$	Joint life n-year pure endowment *Assurance payment payable at time n, provided both lives are still alive.*	$v^n\, {}_np_x\, {}_np_y$
$A_{x:y:\overline{n}\rceil}$	Joint life n-year endowment assurance *Assurance payment made at the end of year of the first death, provided that occur within the next n years, or at time n, provided both lives are still alive.*	$A_{\overset{1}{x:y}:\overline{n}\rceil} + A_{x:y:\overset{1}{\overline{n}\rceil}}$ or $1-d\ddot{a}_{x:y:\overline{n}\rceil}$
$\bar{A}_{x:y:\overline{n}\rceil}$	Joint life n-year endowment assurance *Assurance payment made immediately on the first death, provided that occur within the next n years or at time n, provided both lives are still alive.*	$\bar{A}_{\overset{1}{x:y}:\overline{n}\rceil} + A_{x:y:\overset{1}{\overline{n}\rceil}}$ or $1-\delta\bar{a}_{x:y:\overline{n}\rceil}$

6 Joint Life Annuities

Symbol	Meaning	Calculation Method
$\ddot{a}_{x:y}$	Joint life immediate annuity-due *Annuity payable in advance until the death of the first life, x or y, whenever that occurs*	Using PMA92/PFA92, tabulated at 4%
$\ddot{a}_{x:y:\overline{n}\rceil}$	Joint life n-year temporary immediate annuity-due *Annuity payable in advance until the death of the first life, whenever that occurs, for a maximum of n years*	$\ddot{a}_{x:y} - v^n \, {}_np_x \, {}_np_y \ddot{a}_{x+n:y+n}$
$a_{x:y}$	Joint life immediate annuity *Annuity payable in arrears until the death of the first life, whenever that occurs*	$\ddot{a}_{x:y} - 1$
$a_{x:y:\overline{n}\rceil}$	Joint life n-year temporary immediate annuity *Annuity payable in arrears until the death of the first life, whenever that occurs, for a maximum of n years*	$a_{x:y} - v^n \, {}_np_x \, {}_np_y a_{x+n:y+n}$
$\bar{a}_{x:y}$	Joint life continuously payable immediate annuity *Annuity payable continuously until the death of the first life, whenever that occurs*	$\ddot{a}_{x:y} - 0.5$
$\bar{a}_{x:y:\overline{n}\rceil}$	Joint life n-year temporary immediate annuity continuously payable *Annuity payable continuously until the death of the first life, whenever that occurs, for a maximum of n years*	$\bar{a}_{x:y} - v^n \, {}_np_x \, {}_np_y \bar{a}_{x+n:y+n}$
$\ddot{a}_{\overline{x:y}}$	Last survivor immediate annuity-due *Annuity payable in advance until the death of the second life, whenever that occurs*	$\ddot{a}_x + \ddot{a}_y - \ddot{a}_{x:y}$
$\ddot{a}_{\overline{x:y}:\overline{n}\rceil}$	Last survivor n-year temporary immediate annuity-due *Annuity payable in advance until the death of the second life, whenever that occurs, for a maximum of n years*	$\ddot{a}_{x:\overline{n}\rceil} + \ddot{a}_{y:\overline{n}\rceil} - \ddot{a}_{x:y:\overline{n}\rceil}$
$a_{\overline{x:y}}$	Last survivor immediate annuity *Annuity payable in arrears until the death of the second life, whenever that occurs*	$a_x + a_y - a_{x:y}$
$a_{\overline{x:y}:\overline{n}\rceil}$	Last survivor n-year temporary immediate annuity *Annuity payable in arrears until the death of the second life, whenever that occurs, for a maximum of n years*	$a_{x:\overline{n}\rceil} + a_{y:\overline{n}\rceil} - a_{x:y:\overline{n}\rceil}$
$\bar{a}_{\overline{x:y}}$	Last survivor continuously payable immediate annuity *Annuity payable continuously until the death of the second life, whenever that occurs*	$\bar{a}_x + \bar{a}_y - \bar{a}_{x:y}$
$\bar{a}_{\overline{x:y}:\overline{n}\rceil}$	Last survivor n-year temporary continuously payable immediate annuity *Annuity payable continuously until the death of the second life, whenever that occurs, for a maximum of n years*	$\bar{a}_{x:\overline{n}\rceil} + \bar{a}_{y:\overline{n}\rceil} - \bar{a}_{x:y:\overline{n}\rceil}$
$a_{x\|y}$ & $\ddot{a}_{x\|y}$	Reversionary annuity *Annuity beginning on the death of life x, for the remaining lifetime of life y*	$a_y - a_{x:y}$ $= (\ddot{a}_y - 1) - (\ddot{a}_{x:y} - 1)$ $= \ddot{a}_y - \ddot{a}_{x:y}$ $= \ddot{a}_{x\|y}$

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